A new field variational integrator for simulating dynamics of flexible multibody systems on SE (3)

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EXTENDED ABSTRACT

1 Introduction

The field-theory variational integrators were firstly developed by Marsden et al. [1], which are based on a discrete version of spacetime covariant variational principles. This kind of variational integrators not only can inherit the advantages of classic variational integrators but also provide a systematic way to decouple the spacetime coupling variables by novel interpolation formula [2]. In the classical approaches [3-5], the cross-section configuration variables of geometrically exact beam elements are usually defined on the space $SO(3)\times\mathbb{R}^3$, which implicitly indicates the interpolation of the beam cross-section rotation and translation fields are uncoupled. In this study, based on the work by Shi et al. [7] and the work by Demoures et al. [6], a new field variational integrator (FVI) for simulating flexible multibody dynamics on SE(3) (special Euclidean group) is proposed. The flexible beam of multibody systems are described by geometrically exact beam elements, and the beam cross-section configuration variables are defined on SE(3), which can lead to a naturally coupled representation of beam cross-section translation and rotation fields [6]. The proposed algorithm is based on a discretization of the configuration bundle, Lagrangian density, and covariant variational principle, which can well preserve the system's energy and symplectic structure, especially for the cases requiring long-time simulation. In addition, the independence of the time interpolation and space interpolation of the proposed integrator can be help to improve computation efficiency.

2 Space and time discretization of beam configuration variables

The spacetime discretization is to decompose the time interval $[0, T]$ into N subintervals with time step size Δt, and space interval $[0, L]$ into J subintervals with space step Δs. The configuration of a geometrically exact beam can be described by the node (cross section) configuration variables $\mathbf{H}_{i,j}$. The subscripts $i = 0, \dots, N$ and $j = 1, \dots, J$ respectively denote the integration time step number and element number, and $t_i := i\Delta t$.

The finite element discretization that can preserve the invariance property under the rigid body motions depends on the map τ: $\mathfrak{g}\rightarrow G$. The configuration for an arbitrary cross section of the element at the time node t_i can be determined by the spatial interpolation

$$\mathbf{H}_{i,\,j+a/\Delta s} = \mathbf{H}_{i,\,j}\tau\left(a\left(\hat{\boldsymbol{\eta}}_{i,\,j+1/2} + \hat{\mathbf{E}}_6\right)\right), \tag{1}$$

where $\hat{\bullet}:\ \mathbb{R}^6 \rightarrow \mathfrak{se}(3)$, $\boldsymbol{\eta}_{i,\,j+1/2} \in \mathbb{R}^6$ is the average discrete convective strain between node j and $j+1$ at the time node t_i.

The configuration for an arbitrary cross section of the element at the space node j can be determined by using the temporal interpolation

$$\mathbf{H}_{i+b/\Delta t,\,j} = \mathbf{H}_{i,\,j}\tau\left(b\left(\hat{\boldsymbol{\xi}}_{i+1/2,\,j}\right)\right), \tag{2}$$

where $b\in[0, \Delta t]$, $\boldsymbol{\xi}_{i+1/2,\,j} \in \mathbb{R}^6$ is the average discrete convective velocity between $\mathbf{H}_{i,\,j}$ and $\mathbf{H}_{i+1,\,j}$ in the time interval $[t_i, t_{i+1}]$ of the node j. From Fig. 1 it can be obviously found that the time interpolation and the space interpolation are completely independently, which will significantly reduce the final derived dynamic equations and help to improve computation efficiency.

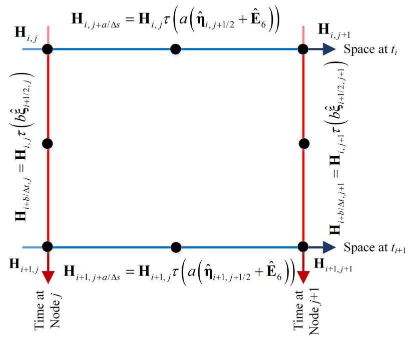

Figure 1: The geometrical view for relations of time and space interpolations.

3 The new field variational integrator on SE(3)

The new field variational integrator for the geometrically exact beam with kinematic constraints can be further expressed by two sets of nonlinear algebraic equations.

$$\begin{bmatrix} \mathbf{P}_{i,1}^{\mathrm{T}} & \cdots & \mathbf{P}_{i,j}^{\mathrm{T}} & \cdots & \mathbf{P}_{i,J+1}^{\mathrm{T}} & \mathbf{\Phi}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = \mathbf{0}, \tag{3}$$

$$\begin{bmatrix} \mathbf{Q}_{i,1}^{\mathrm{T}} & \cdots & \mathbf{Q}_{i,j}^{\mathrm{T}} & \cdots & \mathbf{Q}_{i,J+1}^{\mathrm{T}} & \dot{\mathbf{\Phi}}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = \mathbf{0}, \tag{4}$$

where $\mathbf{P}_{i,j} = \left(\mathbf{I} - \frac{1}{2}\Delta t \mathrm{ad}^{*}_{\xi_{i+1/2,j}}\right)\mathbb{J}\xi_{i+1/2,j} - \mathbb{J}\xi_{i,j} - \frac{1}{2}\Delta t \mathbf{F}_{i,j}^{\mathrm{ext}} + \mathbf{\Phi}_{\mathbf{H}_{i,j}}^{\mathrm{T}} \boldsymbol{\lambda}_i^{-}$, $\mathbf{Q}_{i,j} = -\left(\mathbf{I} + \frac{1}{2}\Delta t \mathrm{ad}^{*}_{\xi_{i+1/2,j}}\right)\mathbb{J}\xi_{i+1,j} + \frac{1}{2}\Delta t \mathbf{F}_{i+1,j}^{\mathrm{ext}} - \mathbf{\Phi}_{\mathbf{H}_{i+1,j}}^{\mathrm{T}} \boldsymbol{\lambda}_{i+1}^{+}$. And $\mathbf{F}_{i,j}^{\mathrm{ext}}$ denotes the discrete external forces, $\mathbf{\Phi} = \mathbf{0}$ is displacement constraint equations, $\dot{\mathbf{\Phi}} = \mathbf{0}$ denotes the velocity constraints and $\boldsymbol{\lambda}_i^{-}$, $\boldsymbol{\lambda}_{i+1}^{+}$ denote Lagrange multipliers.

4 Results

The dynamics of a double pendulum with the initial configuration is simulated. The two arms are respectively meshed by 20 geometrically exact beam elements. The system is released from the initial position with null velocities and under the action of gravity acting in the negative y-direction.

As shown in Fig. 2, the proposed FVI with exponential map and Cay map show an excellent long-time energy conservation property for the studied spatial double pendulum multibody system. We found that for some models with specified parameters the Lie generalized-α algorithm [8] can not preserve the systems' energy well, as shown in Fig. 2.

In addition, the proposed FVI can be further developed by using the concept of frame operator [7] to study the geometrically exact thin-walled beams with warping effects, as shown in Fig. 3. More results will be shown in the conference.

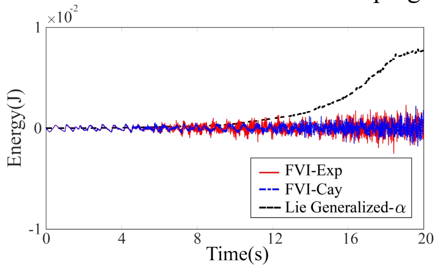

Figure 2: Evolutions of system's total energy

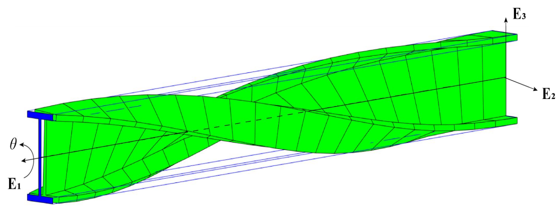

Figure 3: Configurations of a twist I-section cantilever beam

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